

Solutions

EHB322E Digital Electronic Circuits MIDTERM I

Duration: 120 Minutes

Grading: 1) 35%, 2) 35%, 3) 30%

Exam is in closed-notes and closed-books format; calculators are allowed

For your answers please use the space provided in the exam sheet

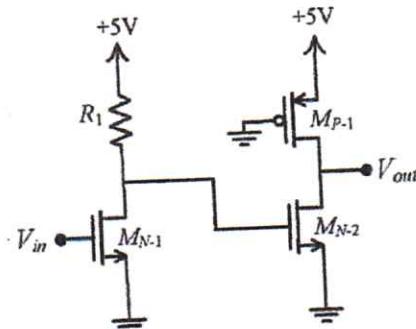
GOOD LUCK!

- 1) Consider a buffer shown below. Use the following equations for your calculations.

Saturation region current-voltage equation: $I_D = \frac{1}{2} k'_{p,n} \frac{W}{L} (V_{GS} - V_{T0,p,n})^2$

Linear region current-voltage equation: $I_D = \frac{1}{2} k'_{p,n} \frac{W}{L} [2(V_{GS} - V_{T0,p,n})V_{DS} - V_{DS}^2]$

Transistor parameters: $k'_p = \mu_p C_{ox} = 54 \mu A/V^2$, $k'_n = \mu_n C_{ox} = 96 \mu A/V^2$, $V_{TN} = 1V$, $V_{TP} = -1V$, $W_{N-1} = W_{N-2} = 12 \mu m$, $L_P = L_N = 1 \mu m$.



Buffer

- Find the minimum value of R_1 if $V_{in}=5V$ results in $V_{out}=5V$.
- Find the value of W_{P-1} if $V_{in}=0V$ results in $V_{out}=0.5V$.
- Find the buffer's static power consumption values when $V_{in}=0V$ and $V_{in}=5V$.
- Using the values found in a) and b), find the value of V_{out} if $V_{in}=2.5V$.

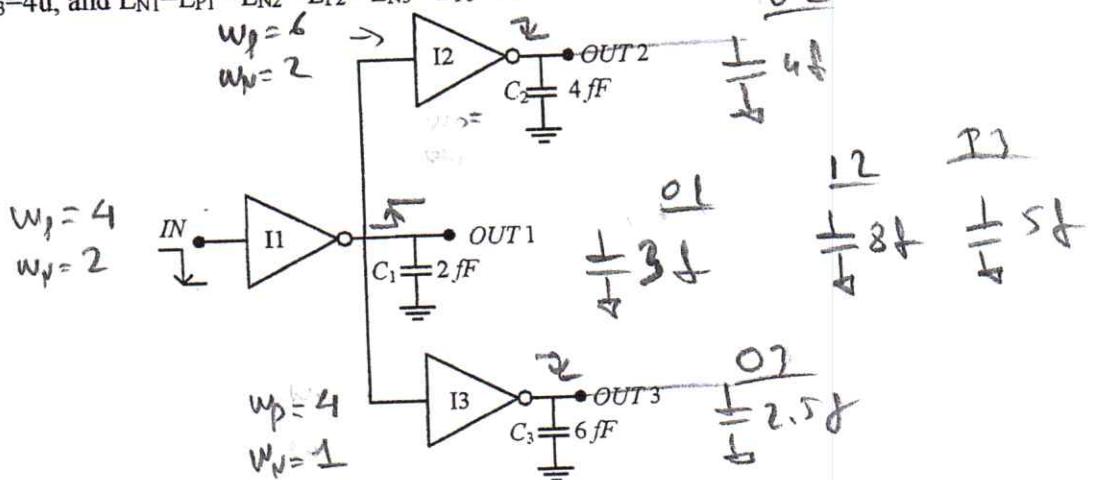
10 c) $V_{GS} = 1V \Rightarrow M_{N-1} \text{ is in linear region}$
 $I_{DN-1} = \frac{1}{2} 96 \cdot 12 \left\{ 2(4) \cdot 1 - 1 \right\} = 4.032 \mu A$
 $\Rightarrow R_1 = (5V - 1V) / I_{DN-1} \approx 1k\Omega$

10 b) $V_{GS} = 5V \Rightarrow M_{N-2} \text{ is in saturation region}$
 $V_{out} = 0.5V \quad M_{P-1} \text{ is in saturation region}$
 $\Rightarrow \frac{1}{2} 96 \cdot 12 \left[2(4) \cdot 0.5 - 0.15 \right] \Rightarrow W_{P-1} = 5 \mu m$
 $I_{DN-2} = 2.16 \mu A$

10 c) $V_{in} = 0V \Rightarrow SP = 2.16 \cdot 5V \approx 10.8 \mu W$
 $\Rightarrow 3.75 = 2.7 \cdot 2 \cdot V_{DS} - V_{DS}^2$
 $V_{in} = 5V \Rightarrow SP = I_{DN-1} \cdot 5V \approx 20 \mu W$
 $\Rightarrow 10.8 = 2.7 \cdot 2 \cdot V_{DS} - V_{DS}^2$
 $\Rightarrow V_{DS} = 0.82$

10 d) Suppose that M_{N-1} is in sat. \Rightarrow Suppose that M_{P-1} is in sat.
 $I_{DN-1} = \frac{1}{2} 96 \cdot 12 (1.1)^2 \approx 1.3 \mu A \Rightarrow V_{DS,N-1} = 3.7V$
 $\Rightarrow V_{out} = 0.82V$

- 2) Consider a circuit with three CMOS inverters and three outputs shown below. External capacitors with values of $2fF$, $4fF$, and $6fF$ are connected to output-1, output-2, and output-3, respectively. A signal switching from high to low is applied to the input. Transistor parameters: $c_{ox} = 1fF/\mu m^2$, $\tau_n = \tau_p = 1ps$, $W_{N1} = 2\mu m$, $W_{P1} = 4\mu m$, $W_{N2} = 2\mu m$, $W_{P2} = 6\mu m$, $W_{N3} = 1\mu m$, $W_{P3} = 4\mu m$, and $L_{N1} = L_{P1} = L_{N2} = L_{P2} = L_{N3} = L_{P3} = 1\mu m$.



Digital circuit with three CMOS inverters

Propagation delays of an inverter are formulated as follows. C_L represents the total (internal and external) load capacitor of an inverter.

$$t_{PHL} = (C_L/C_N) \tau_n \quad C_N = c_{ox} W_N L_N$$

$$t_{PLH} = (C_L/C_P) \tau_p \quad C_P = c_{ox} W_P L_P$$

Suppose that $C_{GS-N} = C_N$, $C_{GS-P} = C_P$, and each inverter has an internal input capacitor of $(C_{GS-N} + C_{GS-P})$.

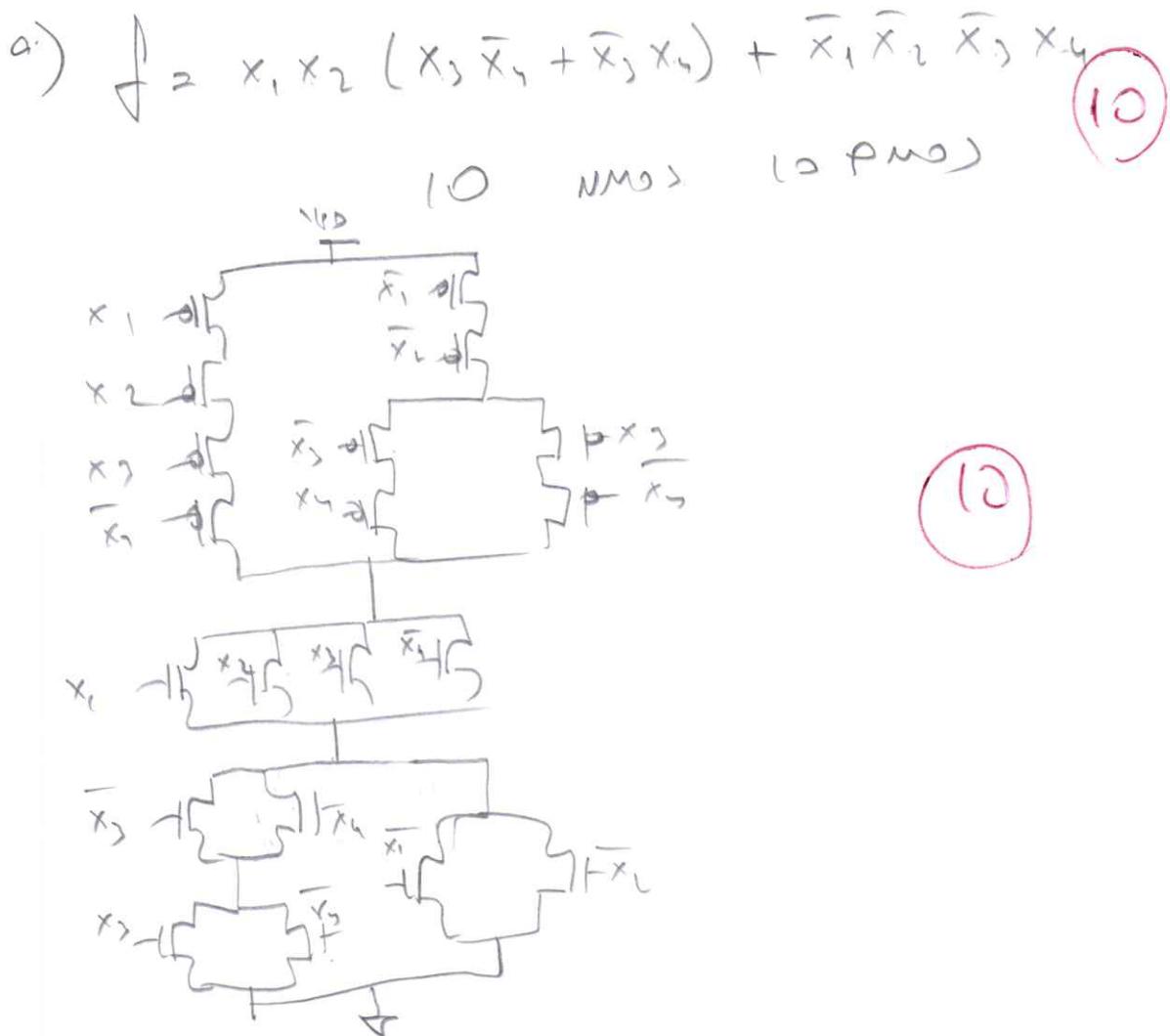
- a) Neglect the inverters' internal output capacitors and find **total propagation delay values** at output-1, output-2, and output-3.
b) Suppose that each inverter has an output internal capacitor $C_{I-out} = c_{ox}(W_N + W_P)(0.5\mu m)$. Find **total propagation delay values** at output-1, output-2, and output-3.

for writing the equations

a) $t_{PD-1} = t_{PLH1} = \frac{C_L}{C_P} \tau_p \quad C_L = 2f + 8f + 5f = 15f$
 $C_P = 4f \quad \Rightarrow t_{PD-1} = \frac{15}{4} 1ps = 3.75ps$
 $t_{PD-2} = t_{PLH1} + t_{PHL2} \quad t_{PHL2} = \frac{C_L}{C_N} \tau_n \Rightarrow t_{PHL2} = 2ps$
 $C_L = 4f \quad C_N = 2f$
 $t_{PD-3} = t_{PLH1} + t_{PHL3} \quad t_{PHL3} = \frac{6}{1} ps$
 $t_{PD-3} = 9.75ps$

b) $t_{PD-1} = t_{PLH1} = \frac{2+3+8+5}{4} ps = 4.5ps$
 $t_{PD-2} = t_{PLH1} + t_{PHL2} = 4.5ps + \frac{8}{2} ps = 8.5ps$
 $t_{PD-3} = t_{PLH1} + t_{PHL3} = 4.5ps + \frac{8.5}{1} ps = 13ps$

- 3) Consider $f = x_1x_2x_3\bar{x}_4 + x_1x_2\bar{x}_3x_4 + \bar{x}_1\bar{x}_2\bar{x}_3x_4$.
- Implement f with a **CMOS circuit** using **minimum** number of transistors. Draw the circuit. How many PMOS and NMOS transistors do you use?
 - Suppose that both NMOS and PMOS transistors have equivalent resistance values of $1k\Omega$; a total output load capacitor is $2fF$ (Neglect all other internal capacitors). Find the **worst case (largest) t_{PHL} and t_{PLH} values.**



b)

wc $t_{PHL} = 0.63(3 \times 1k\Omega)(2f) = 4.14ps$

wc $t_{PLH} = 0.63(4 \times 1k\Omega)(2f) = 5.52ps$

(10)