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Student ID:

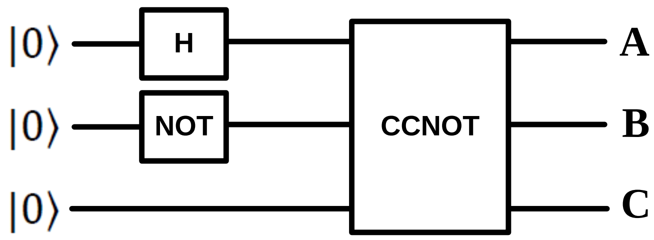
ELE523E Computational Nanoelectronics
Homework 1

Deadline: 1/11/2021 (before 13:30)

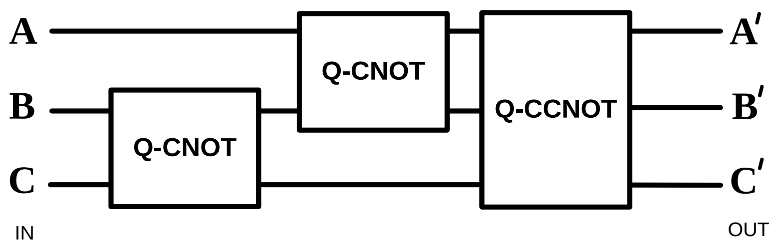
1. QUANTUM COMPUTING

a) For the quantum circuit shown below, find the output quantum state and determine the probabilities of each output combination.

- Hint: the output state can be formalized as $\sum_i \alpha_i |(ABC)_i\rangle$; you need to find $\boldsymbol{\alpha}$ values.



b) Find the truth table of the quantum circuit shown below.



c) Prove that Toffoli (CCNOT) gate is a universal quantum gate (in order to implement any Boolean function).

- Hint: try to implement main Boolean operators
- You are allowed to use $|0\rangle$ or $|1\rangle$ as gate inputs.

d) Implement the Boolean function $f = x_1x_2 + x_1x_3 + x_2x_3x_4$ using *minimum* number of Toffoli gates.

- You are allowed to use $|0\rangle$ or $|1\rangle$ as gate inputs.

2. REVERSIBLE CIRCUIT DESIGN

a) Implement the following truth tables with reversible circuits using NOT, CNOT, and Toffoli (CCNOT) gates.

IN cba	OUT cba
000	001
001	111
010	000
011	110
100	101
101	100
110	011
111	010

IN cba	OUT cba
000	111
001	001
010	011
011	010
100	110
101	000
110	100
111	101

b) Implement the following truth table with a reversible circuit using *minimum* number NOT and CNOT gates.

IN	**OUT**
ba	**ba**
00	11
01	01
10	10
11	00

c) Determine the number of input/output bits of a reversible binary multiplier transformed from a 2-bit by 2-bit irreversible multiplier.

d) Determine the number of input/output bits of a reversible binary multiplier transformed from a 3-bit by 3-bit irreversible multiplier.

3. FACTORIZING SEMI-PRIME NUMBERS

a) Write an algorithm that factorizes *semi-prime numbers*.

- Attach your pseudo and real codes (Matlab, C, etc.) to your homework.

b) Determine *the worst-case time complexity* of your algorithm in terms of the number of digits. Is it polynomial?

c) Determine the success rate of your algorithm. Does it always give you the right answer?

d) To evaluate your algorithm's performance, use semi-prime numbers **15**, **77**, **529**, and **4633** as inputs. Determine the running time of your algorithm for each case.

Grading: 1a)5%, 1b)5%, 1c)10%, 1d)10%
2a)10%, 2b)10%, 2c)7.5%, 2d)7.5%,
3a)15%, 3b)10%, 3c)5%, 3d)5%

Note: *Submit your hardcopies before the lecture.*