HW1 Solutions

1) a) $(100111,10111)_2 = 2^5+2^2+2^1+2^0+2^{-1}+2^{-3}+2^{-4}+2^{-5} = (39,71875)_{10}$

$(\underbrace{100}_{4}\underbrace{111}_{7},\underbrace{101}_{5}\underbrace{110}_{6})_2 = (47,56)_8$

$(\underbrace{10}_{2}\underbrace{0111}_{7},\underbrace{1011}_{B}\underbrace{1000}_{8})_2 = (27,B8)_{16}$

b) $(7\,2,6)_8$ (111 010 011) $= (\underbrace{11}_{3}\underbrace{1010}_{A},\underbrace{1100}_{C})_2 = (3A,C)_{16} = 7.8^1+2.8^0+6.8^{-1} = (58,75)_{10}$

c) $(C3,AD5)_{16} = (\underbrace{1100\,0011}_{3\;0\;3},\underbrace{101011010101}_{5\;3\;2\;5})_2 = (303,5325)_8 = 12.16^1+3.16^0+10.16^{-1}+13.16^{-2}+5.16^{-3}$

1100 0011 1010 1101 0101

$= (195,677001953125)_{10}$

2) a) $\overline{x_1x_2+x_2x_3+x_3x_4} = (\bar{x}_1+\bar{x}_2)(\bar{x}_2+\bar{x}_3)(\bar{x}_3+\bar{x}_4)$

$= \bar{x}_1\bar{x}_2\bar{x}_3+\bar{x}_1\bar{x}_2\bar{x}_4+\bar{x}_1\underline{\bar{x}_3\bar{x}_3}+\bar{x}_1\bar{x}_3\bar{x}_4+\underline{\bar{x}_2\bar{x}_2}\bar{x}_3+\underline{\bar{x}_2\bar{x}_2}\bar{x}_4+\bar{x}_2\underline{\bar{x}_3\bar{x}_3}+\bar{x}_2\bar{x}_3\bar{x}_4$

$= \cancel{\bar{x}_1\bar{x}_2\bar{x}_3}+\cancel{\bar{x}_1\bar{x}_2\bar{x}_4}+\bar{x}_1\bar{x}_3+\cancel{\bar{x}_1\bar{x}_3\bar{x}_4}+\bar{x}_2\bar{x}_3+\bar{x}_2\bar{x}_4+\cancel{\bar{x}_2\bar{x}_3}+\cancel{\bar{x}_2\bar{x}_3\bar{x}_4}$

$= \bar{x}_1\bar{x}_3+\bar{x}_2\bar{x}_3+\bar{x}_2\bar{x}_4 \Rightarrow$ 6 literals

b) $\overline{\bar{x}_1x_2x_3+x_1x_4} = (x_1+\bar{x}_2+\bar{x}_3)(\bar{x}_1+\bar{x}_4) = \cancel{x_1.\bar{x}_1}+x_1\bar{x}_4+\bar{x}_2\bar{x}_1+\cancel{\bar{x}_2\bar{x}_4}+\bar{x}_1\bar{x}_3+\cancel{\bar{x}_3\bar{x}_4}$

$= x_1\bar{x}_4+\bar{x}_1\bar{x}_2+\bar{x}_1\bar{x}_3 \Rightarrow$ 6 literals

c) $\overline{x_1\bar{x}_2x_3+x_1\bar{x}_4+x_2x_3\bar{x}_4} = (\bar{x}_1+x_2+\bar{x}_3)(\bar{x}_1+x_4).(\bar{x}_2+\bar{x}_3+x_4)$

$= \cancel{\bar{x}_1}\bar{x}_1\bar{x}_2+\cancel{\bar{x}_1}\bar{x}_1\bar{x}_3+\cancel{\bar{x}_1}\bar{x}_1x_4+\bar{x}_1x_4\bar{x}_2+\bar{x}_1x_4\bar{x}_3+\bar{x}_1x_4\cancel{x_4}+\cancel{x_2\bar{x}_1\bar{x}_2}+x_2\bar{x}_1\bar{x}_3+x_2\bar{x}_1x_4+\cancel{x_2x_4\bar{x}_2}+x_2x_4\bar{x}_3+x_2x_4\cancel{x_4}$

$+\bar{x}_3\bar{x}_1\bar{x}_2+\bar{x}_3\bar{x}_1\cancel{\bar{x}_3}+\bar{x}_3\bar{x}_1x_4+\bar{x}_3x_4\bar{x}_2+\bar{x}_3x_4\cancel{\bar{x}_3}+\bar{x}_3x_4\cancel{x_4}$

$= \bar{x}_1\bar{x}_2+\bar{x}_1\bar{x}_3+\bar{x}_1x_4+\cancel{\bar{x}_1\bar{x}_2x_4}+\cancel{\bar{x}_1\bar{x}_3x_4}+\cancel{\bar{x}_1x_4}+\cancel{\bar{x}_1x_2\bar{x}_3}+\cancel{\bar{x}_1x_2x_4}+\cancel{x_2\bar{x}_3x_4}+x_2x_4+\cancel{\bar{x}_1\bar{x}_2\bar{x}_3}+\cancel{\bar{x}_1\bar{x}_3}+$

$\cancel{\bar{x}_1\bar{x}_3}x_4+\cancel{\bar{x}_2\bar{x}_3}x_4+\bar{x}_3x_4$

$= \bar{x}_1\bar{x}_2+\bar{x}_1\bar{x}_3+\cancel{\bar{x}_1x_4}+x_2x_4+\bar{x}_3x_4 = \bar{x}_1\bar{x}_2+\bar{x}_1\bar{x}_3+x_2x_4+\bar{x}_3x_4 \Rightarrow$ 8 literals

2- d) $\overline{x_1 x_2 \bar{x}_3 + x_1 \bar{x}_2 x_3 + \bar{x}_1 x_2 x_3 + \bar{x}_1 \bar{x}_2 \bar{x}_3} = \overline{x_1 (\underbrace{x_2 \bar{x}_3 + \bar{x}_2 x_3}_{a}) + \bar{x}_1 (\underbrace{x_2 x_3 + \bar{x}_2 \bar{x}_3}_{\bar{a}})}$

$= \overline{x_1 \cdot a + \bar{x}_1 \cdot \bar{a}} = (\bar{x}_1 + \bar{a})(x_1 + a) = \cancel{\bar{x}_1 x_1} + \bar{x}_1 a + a x_1 \cancel{+ \bar{a} a}$

$= \bar{x}_1 (x_2 \bar{x}_3 + \bar{x}_2 x_3) + x_1 (x_2 x_3 + \bar{x}_2 \bar{x}_3)$

$= \bar{x}_1 x_2 \bar{x}_3 + \bar{x}_1 \bar{x}_2 x_3 + x_1 x_2 x_3 + x_1 \bar{x}_2 \bar{x}_3 \Rightarrow$ 12 literals

3-)

X_1	X_2	X_3	X_4	Out
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

$\Rightarrow$ assigned arbitrarily

Each arrow corresponds to a transition in just one of the inputs.

4-

SOP FORM:

$$C_{out} = \bar{A}BC_{in} + A\bar{B}C_{in} + AB\overline{C_{in}} + ABC_{in}$$

$$C_{out} = C_{in}(\bar{A}B + A\bar{B}) + AB\underbrace{(\overline{C_{in}} + C_{in})}_{1}$$

$$C_{out} = C_{in} \cdot (A \oplus B) + AB$$

$$S = \bar{A}\bar{B}C_{in} + \bar{A}B\overline{C_{in}} + A\bar{B}\overline{C_{in}} + ABC_{in}$$

$$S = C_{in} \cdot (\bar{A}\bar{B} + AB) + \overline{C_{in}}(\bar{A}B + A\bar{B})$$

Let's say; $\bar{A}B + A\bar{B} = D = A \oplus B$

$\bar{A}\bar{B} + AB = \bar{D}$

$$S = C_{in}\bar{D} + \overline{C_{in}}D = C_{in} \oplus D$$

$$S = C_{in} \oplus (A \oplus B)$$

POS FORM:

$$C_{out} = (A + B + C_{in}) \cdot (A + B + \overline{C_{in}}) \cdot (A + \bar{B} + C_{in}) \cdot (\bar{A} + B + C_{in})$$

$$C_{out} = \left[A + B + (C_{in} \cdot \overline{C_{in}})\right] \cdot \left[C_{in} + (A + \bar{B}) \cdot (\bar{A} + B)\right]$$

$$C_{out} = (A + B) \cdot \left[C_{in} + \overline{A \oplus B}\right]$$

$$S = (A+B+C_{in}) \cdot (A+\overline{B}+\overline{C_{in}}) \cdot (\overline{A}+B+\overline{C_{in}}) + (\overline{A}+\overline{B}+C_{in})$$

$$S = \left[C_{in} + \underbrace{(A+B) \cdot (\overline{A}+\overline{B})}_{A \oplus B}\right] \cdot \left[\overline{C_{in}} + \underbrace{(A+\overline{B}) \cdot (\overline{A}+B)}_{\overline{A \oplus B}}\right]$$

$$S = (C_{in} + A \oplus B) \cdot (\overline{C_{in}} + \overline{A \oplus B})$$

$$S = C_{in} \oplus (A \oplus B)$$

5

$$\begin{array}{rrrr} & & a_1 & a_0 \\ \times & & b_1 & b_0 \\ \hline & & (a_1 b_0) & (a_0 b_0) \\ + & (a_1 b_1) & (a_0 b_1) & \\ \hline m_3 & m_2 & m_1 & m_0 \end{array}$$

Output $= m_3 m_2 m_1 m_0$