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A change-point based reliability prediction model using field return data ☆

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ABSTRACT

In this study, we propose an accurate reliability prediction model for high-volume complex electronic products throughout their warranty periods by using field return data. Our model has a specific application to electronics boards with given case studies using 36-month warranty data. Our model is constructed on a Weibull-exponential hazard rate scheme by using the proposed change point detection method based on backward and forward data analysis. We consider field return data as short-term and long-term corresponding to early failure and useful life phases of the products, respectively. The proposed model is evaluated by applying it to four different board data sets. Each data set has between 1500 and 4000 board failures. Our prediction model can make a 36-month (full warranty) reliability prediction of a board with using its field data as short as 3 months. The predicted results from our model and the direct results using full warranty data match well. This demonstrates the accuracy of our model. We also evaluate our change point method by applying it to our board data sets as well as to a well-known heart transplant data set.

1. Introduction

The rapid developments in electronics, especially in the last decade, have elevated the importance of electronics reliability [2–4]. Complex electronic systems, spanning almost all large industrial fields, require high reliability that necessitates accurate and early reliability predictions to give feedback for design and warranty precautions. Prediction methods in the literature are mainly based on accelerated life tests, component based numerical and probabilistic simulations, and statistical field data analysis [2,5–7]. Despite their widely usage, accelerated tests do not meet the needs of today's very rapid electronic product cycles; they are time consuming and expensive [8]. To overcome this problem, tests can be supported by simulations. However, in general simulations have severe accuracy limitations especially for complex electronic systems having various failure mechanisms [2,8,9]. Therefore, accelerated test and simulation based predictions can be deceptive for many applications. This underlines the importance of using field return data for reliability prediction that is relatively accurate, cheap, and time saving.

In this study, we perform warranty forecasting of high-volume complex electronic products using their field return data. We have cooperated with one of the Europe's largest household appliance manufacturers and used their well-maintained data sets of four different electronic boards. Each data set has between 1500 and 4000 board failures.

We construct our prediction model on a Weibull-exponential piecewise hazard rate scheme by considering early failure and useful life phases of the products. This scheme is preferred for products not getting into the wear-out region during their warranty times or anticipated lives [10,11]. Electronic products including electronic boards thoroughly studied in this work, fall into these product groups. As shown in Fig. 1, electronic boards are expected to work at least 10–15 years with a warranty period of at most 5 years.

Using a piecewise hazard rate function for our reliability model necessitates to determine the turning/change time distinguishing Weibull and exponential distributions. This problem, often called as the change point problem in the literature, is of fundamental importance in various applications including bio-statistics and medical survival data analysis [13–15]. Different approaches have been proposed to solve the problem [16]. Studies exploiting parametric change point analysis of non-monotonic hazard rate functions consider the change point as a parameter and propose statistical estimation methods including maximum likelihood, least squares, and Bayesian methods [15,17,18]. Additionally, nonparametric methods are studied widely [13,19].

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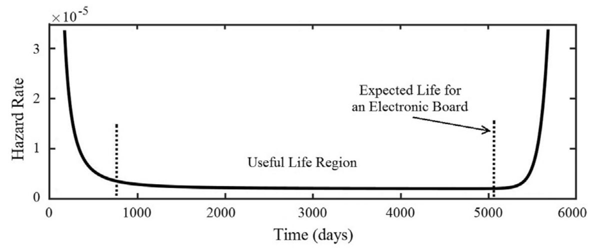

Fig. 1. A bathtub curve – hazard rate function over time [12].

Although there are both parametric and non-parametric methods in the literature giving theoretically satisfactory results for the change point problem, they are loosely encountered in engineering areas in the manner of applicability. They are usually evaluated using certain data sets with certain distributions such as biological survival data having exponential-exponential hazard rate scheme [15,16,18]. However, these methods can be deceptive for unevenly distributed data in time domain that needs different approaches and distributions for different time spans. For example, consider our board data sets for which nearly 90% and 10% of the data belong to the early failure (Weibull distribution) and the useful life (exponential distribution) regions, respectively. With the above discussed methods, the change point that separates the regions can not be determined accurately. Motivated by this, we propose a practical yet accurate change point detection method by performing forward and backward data analysis with left and right truncated data. We exploit both parametric and non-parametric techniques by using maximum likelihood and rank regression estimations.

Constructed on the proposed change point detection method, we develop an early reliability prediction methodology. In our case studies, we accurately predict 36-month reliability of electronic boards by using their 3-month field data. Investigating the related studies in the literature, we see the inappropriateness of non-parametric methods since they can only predict near future reliability of a new product [20–23]. Parametric methods using time dependent parameters are more suitable in this regard. In general, parametric methods including a standard Bayesian estimation, use a prior parameter distribution assuming that the distribution is same for an old product from which we get prior knowledge and a new product for which we perform reliability estimation [24–27]. This assumption is satisfied if the parameter is defined for a specific failure in a specific material/component. However, for complex systems having various components including electronic products targeted in this study, the assumption looses its validity.

There are also studies using empirical findings to find distributions of the parameters [28,29]. Here, the main problem is that to obtain sufficient accuracy, high amount of data is needed for a new product that can kill the idea of early prediction. Some recent works aim to solve this problem by using data from pass-fail reliability tests and having field data from multiple products [30–32]. In Ref. [31], field data of 17 different products are used for early prediction with an assumption that these products have similar failure distributions. Another solution is performing degradation tests and condition monitoring to have more data [33–35]. Of course, these techniques are considerably costly compared to the techniques, including ours, that just use field data. Additionally, physics of failure based simulations are exploited for reliability prediction [36]. Although this technique gives satisfactory results for individual components having certain failure mechanisms, its accuracy is susceptible for systems having hundreds of components including electronic products targeted in this study.

As opposed to the mentioned studies in the literature, our reliability prediction model uses only field data and deals with a single new product. Our model combines predetermined parameter distributions and empirical findings, and performs data fitting. It has an input of either short-term or long-term field data corresponding to early failure and useful life phases of products, respectively. The output of the model is the reliability prediction covering the full warranty period. In case of having long-term data, we directly apply our change point method to depict a hazard rate curve with Weibull and exponential distributions using maximum likelihood and rank regression methods. In case of having short-term data, we first investigate how the Weibull shape parameter β varies with different time intervals of the field data. For this purpose, we use full warranty field data belonging to previous and/or current versions of the products. We develop a mathematical function of β in terms of time and product dependent parameters. Additionally, we use empirical findings and iteration methods to estimate other distribution parameters. We finally construct our prediction model based on the developed Weibull-exponential scheme.

The paper is organized as follows. In Section 2, we introduce our reliability prediction methodology with a flowchart. In the following sections we elaborate on the flowchart step by step. In Section 3, we represent our forward and backward data analysis method to achieve a Weibull-exponential piecewise hazard rate scheme. We evaluate our method by applying it to our field data as well as to a well-known heart transplant data. In Section 4, we represent our reliability prediction model with long-term and short-term data. The proposed model is evaluated by applying it to different electronic board families. In Section 5, conclusion remarks and inferences about future works are given.

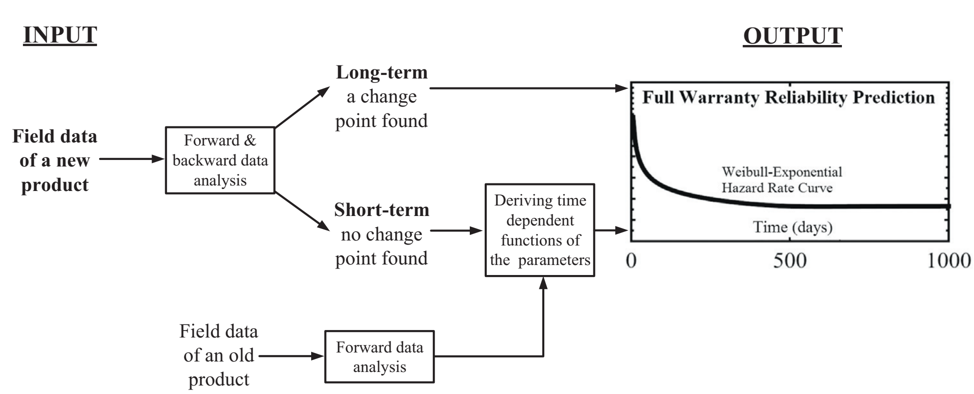

Fig. 2. Flow of the proposed reliability prediction methodology.

2. The proposed methodology

Our methodology, illustrated in Fig. 2, has an input of a new product's field data classified as long-term or short-term that is determined by applying the proposed forward and backward data analysis. If the analysis results in a change point that separates early failure and useful life regions then we call the input data as long-term data that is modelled with Weibull and exponential distributions. In this case, reliability prediction is achieved by using an exponential distribution that is expected to keep its validity until the end of the warranty period. If the analysis results in a single distribution, no change point found, then we call the input data as short-term data. Here, we use a framework that connects prior knowledge based on an old product with a new one. We evaluate the field data of an old product in forward time spans in order to obtain time dependent behaviour of the Weibull β parameter.

Ideally, a specific component or a material is expected to have its own constant β that is time independent. However, similarly for most of the complex systems, electronic products consist of hundreds of different components/elements. Some of these elements such as integrated chips are almost perfectly reliable throughout the early failure phase that results in $\beta > 1$. On the other hand, components, especially used in power supply and mechanical control blocks which face much more stress, have serious early reliability issues that results in $\beta < 1$. Therefore it is expected to see changes for β along with time to failure values. This is our main motivation to derive a time dependent function of β.

From empirical findings, we first determine the function form of β by considering data sets of old products. Determination is not purely based on statistical inferences, but also has an engineering point of view. From our case studies using four different board data sets, we claim that complex electronic products have a logarithmic β function with two product dependent parameters. While one of the parameters has a fixed value for all products in the same family such as boards of washing machines, the other one varies for each product. We estimate these parameters using a shrinkage technique by considering root-mean-square error values. Note that we have a predetermined distribution with unknown parameters to be estimated that is different than both empirical and standard Bayesian estimation techniques. After estimating parameter values, we directly use the value of the family dependent parameter for a new product in the same family. Then we estimate the other parameter using the new product's data. For this estimation, we use a maximum likelihood estimation (MLE) method regarding large sample sizes of high-volume products. However, in our previous study we show that Bayesian estimation would give better results in case of having smaller data sets; the difference is obvious for sample sizes in the order of tens [1]. To estimate other parameters in the Weibull-exponential scheme, we use empirical findings and iteration methods. As a result, we make a full warranty forecasting of a new product given its short-term field data.

Summary of the proposed methodology is given in Fig. 3. Step 1 and Step 2 of the methodology are thoroughly explained in Sections 3 and 4, respectively. In these sections, the steps are explicitly evaluated by using four different board data sets. As follows, we give information about these sets.

Input: Field data of a new product not completing its warranty period.

Goal: Full warranty reliability prediction with hazard rate curves.

Step 1 **Chance point detection**: Apply maximum likelihood and rank regression methods for data fitting while enhancing time windows in forward and backward directions.

Step 2 **Reliability prediction**: Using full warranty data of an old product, estimate the parameters of a new product using a shrinkage technique, maximum likelihood method, and empirical findings.

Fig. 3. Outline of the proposed reliability prediction methodology.

2.1. Board data sets

We use four different data sets belonging to four different boards Board-B, Board-E, Board-F, and Board-K. The field data contain assembly and return/failure dates for each warranty call – time differences between the dates give time to failure values. Warranty period of the boards is 3 years. The maintenance policy of the company is to replace the failed board with a new one in any suspected case such as stopping from time to time or breaking down entirely in the field. Therefore the data sets have no record of repaired boards. All failed boards are inspected to find root causes of the failures by the company. While most of the failures are linked to their root causes, there are also cases where no cause is found and the replaced boards work properly during inspections. These cases are disregarded in this study to improve data accuracy. In other words, we only consider proven failures. To further improve the data accuracy, we apply our “filtering technique” that eliminates errors such as data losses and inappropriate data recordings [37]. A summary of the final form of the data is given in Table 1. The collection was performed from the first day of products in field.

3. Change point detection by forward and backward data analysis

As we previously discussed in the introduction part, field return data of electronic products can not be used for analysis of the wear-out region since they get into this region long after their warranties expire. Therefore, we only deal with early failure and useful time periods. We develop our hazard rate function with phases having a decreasing failure rate (DFR) and a constant failure rate (CFR). Here, DFR and CFR correspond to early failure and useful life regions, respectively. The overall hazard rate function $h_o(t)$ is defined as

$$h_o(t) = \begin{cases} h_1(t), & t < t_c \\ h_2(t), & t > t_c \end{cases} \tag{1}$$

where $h_1(t)$ and $h_2(t)$ refer to hazard rate functions having DFR and CFR, respectively, and t_c refers to a change point distinguishing DFR and CFR. Note that it is not necessary to achieve continuity between $h_1(t)$ and $h_2(t)$. Additionally after finding a change point t_c, $h_1(t)$ and $h_2(t)$ are generally obtained using separate data sets as we do in Section 3.2. As a result, there is no direct dependency between $h_1(t)$ and $h_2(t)$. Considering a discontinuous $h_o(t)$ at t_c, Eq. (1) can be written as Eq. (2) with an addition of an indicator function $I(t)$.

$$h_o = I(t \le t_c)h_1(t) + I(t > t_c)h_2(t). \tag{2}$$

Constructed on Eq. (2), we first derive formulations for the

Table 1
Summary of the board data sets.

Board name	Total number of products in field	Total number of failures	Duration of data collection (months)
B	689,536	1849	37
E	1,048,201	4050	40
F	286,108	1533	24
K	70,852	3936	38

conventional MLE method in order to find a change point. We see that the obtained change point values for board data sets do not adequately meet our expectations. Next, we present our change point method based on forward and backward data analysis using both MLE and rank regression estimation (RRE) methods. We successfully apply our method to the data sets as well as to the medical failure data.

3.1. Pseudo maximum likelihood estimation

General formulations for a mixed hazard rate model are given below [14].

$$H_o = I(t \le t_c)H_1(t) + I(t > t_c)[H_1(t_c) + H_2(t) - H_2(t_c)] \quad (3)$$

$$g(t) = \left(1 - e^{-H_1(t_c)}\right)f_1(t) + e^{-H_1(t_c)}f_2(t) \quad (4)$$

$$f_1(t) = \frac{h_1(t)e^{-H_1(t)}}{1 - e^{-H_1(t_c)}}I(t \le t_c) \quad (5)$$

$$f_2(t) = h_2(t)e^{-(H_2(t) - H_2(t_c))}I(t > t_c) \quad (6)$$

$$S(t) = \frac{e^{-H_1(t)} - e^{-H_1(t_c)}}{1 - e^{-H_1(t_c)}}I(t \le t_c) + e^{-(H_2(t) - H_2(t_c))}I(t > t_c) \quad (7)$$

Here, $H(t)$ represents a cumulative hazard rate function of $h(t)$. The functions $g(t)$, $f_1(t)$, and $f_2(t)$ are overall mixture, right truncated, and left truncated density functions, respectively; $S(t)$ denotes a survival function.

In the Weibull-exponential mixture scheme, the right truncated density function in Eq. (5) and the left truncated density function in Eq. (6) are Weibull and exponential density functions, respectively. A likelihood function for right censored data, encountered in warranty returns, is given as

$$LL = \prod g(ti)_i^{w} S(ti)^{1-w_i} \quad (8)$$

where $g(t)$, an overall mixture density function derived using Eqs. (5) and (6), is formulated as

$$g(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} e^{-\frac{t}{\eta}^{\beta}} I(t \le t_c) + \lambda e^{-(t_c/\eta)^{\beta}} e^{-(\lambda t - \lambda t_c)} I(t > t_c). \quad (9)$$

In Eq. (8), w_i takes a value of 1 and 0 in case of failure and suspended cases, respectively. There are four unknown parameters λ, η, β, and t_c to be estimated in this scheme. However, according to Eq. (8), t_c can not be directly estimated since there is a discontinuity at t_c. Additionally, it is explicitly shown that it is hard to get an inference for t_c except for exponential-exponential mixture scheme [38]. Considering these issues, we use a pseudo MLE approach [39]. We do not purely use algebraic approaches that is computationally intractable regarding the four unknown parameters. Instead, we perform a grid search for t_c assuming that we know other parameters. We use values obtained from single distribution fittings for other parameters in every iteration of t_c.

We apply our pseudo MLE method to the data sets of the boards Board-B, Board-E, Board-F, and Board-K. Results are given in Table 2 with numbers representing average likelihood function values $log(LL)/(samplesize)$ that are calculated using Eq. (8). We choose grid time spans as 60 days. Although smaller grid spans are expected to result in more accurate and detailed results due to an asymptotic behaviour of MLE, in our case narrowing them beyond 60 days does not give us any valuable inference. Therefore, for simplicity we use 60-day time spans. Note that total time range is 1080 days (36-month full warranty) for all boards except for Board-F that has a data collection duration of 720 days.

Examining the numbers in Table 2, we can confidently make a change point prediction for the data set of Board-K for which the likelihood values first increase, then stabilize for a short time, and finally decrease. This is a desired phenomena to successfully determine the change point t_c that should be in the time range of stabilized values. As a result, the change point for Board-K is around 480 days. For Board-F, likelihood ratio increases up to 660 days. Therefore, the change point is probably around 660 days. On the other hand, for boards B and E no strong inferences can be obtained since likelihood values excessively fluctuate. While the presumed t_c value for Board-F is consistent with the result of our method given in the next section, that for Board-K is not consistent with the result of our method. More accurate results could be achieved by extending grid searches, say for both Weibull β and t_c. However, this would exponentially increase the computing time that is impractical for large data sets. Regarding these problems, we propose a practical yet accurate change point detection method in the following section.

Table 2
Likelihood values for different time intervals.

t (days)	Board-B	Board-E	Board-F	Board-K
60	−0.129	−0.221	−0.0676	−0.731
120	−0.230	−0.228	−0.0663	−0.652
180	−0.340	−0.220	−0.0653	−0.673
240	−0.252	−0.227	−0.0645	−0.342
300	−0.437	−0.322	−0.0638	−0.296
360	−0.358	−0.301	−0.0634	−0.295
420	−0.394	−0.485	−0.0632	−0.223
480	−0.522	−0.497	−0.0631	−0.278
540	−0.638	−0.432	−0.0623	−0.322
600	−0.775	−0.421	−0.0631	−0.327
660	−0.854	−0.423	−0.0631	−0.329
720	−0.954	−0.417	−0.429	−0.414
780	−0.955	−0.422	–	−0.414
840	−0.955	−0.425	–	−0.417
900	−0.958	−0.434	–	−0.419
960	−0.974	−0.456	–	−0.423
1020	−0.974	−0.500	–	−0.464
1080	−0.974	−0.500	–	−0.464

3.2. Proposed change point estimation

In our method, we make data analysis using forward and backward time windows. We apply MLE and RRE methods for data fitting while enhancing time windows in forward and backward directions that can be considered as on-line (adaptive) analysis. For small sample sizes in the order of tens, we use the RRE method; otherwise we prefer the MLE method. While forward analysis is a widely used method in reliability and warranty analysis, investigating time to failure (TTF) values in backward direction is a new method that we introduce for accurate determination of the change point t_c and the hazard functions $h_1(t)$ and $h_2(t)$. In our method, M_f and M_b are defined as the number of months that constitute the boundaries of time windows. Fig. 4 illustrates our method with an explicit demonstration of the time windows that are expanded between 1 and 36 months assuming that our products have a 3-year warranty. Note that a unit time change of 1 month (30 days) is not a necessity; for different applications it can be increased or decreased. Also note that in case of having limited data not covering the full warranty period, time limits need to be updated. For example, Board-F has 24-month data, so time windows can be expanded between 1 and 24 months. To show this, we explicitly apply our method to the data sets of Board-F along with Board-B.

The forward analysis is conducted by using the field return data with TTF values less than or equal to M_f. For example, if $M_f = 3$, the data to be analysed will contain TTF values of 1, 2, and 3 months.

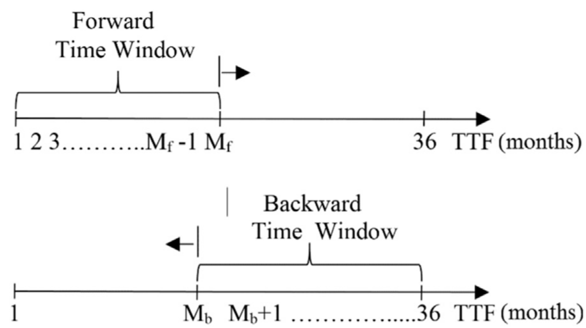

Fig. 4. Demonstration of forward and backward data analysis.

After starting with $M_f = 1$, M_f is increased by adding months one-by-one ($M_f = 2, 3, 4, \ldots, 36$). Therefore, the forward time window is gradually expanding to the end of the TTF line as seen in the upper part of Fig. 4. For each time window, we use different distributions and test them for goodness of fit.

In order to elucidate the forward analysis we show detailed results for Board-B and Board-F. We use a software Reliasoft-Weibull++ [40] to obtain MLE likelihood values $log(LL)/(samplesize)$ that are given in Tables 3, 4. Examining the numbers, we see that a Weibull distribution is the best fit for almost all different M_f values. Although in some cases the best fitting is achieved with lognormal or gamma distributions, these distributions show almost the same DFR pattern as that of a Weibull distribution. Therefore we select a Weibull distribution for $h_1(t)$. Note that in the forward analysis we do not see a change point for M_f from which the DFR pattern and/or the fitting changes significantly. The reason is that the overwhelmingly large portion of failures is gathered at the first six months of the warranty period (TTF $\leq$ 6 months) that dominates the DFR pattern.

The backward analysis is conducted by using TTF values between M_b and 36 months (1–36, 2–36,…,30–36,…). In other words, one end of the backward time window is fixed at the 36^{th} month and the other is gradually expanding to the beginning of the TTF line as seen in the lower part of Fig. 4.

In order to elucidate the backward analysis we show detailed results for Board-B and Board-F. We use a software Reliasoft-Weibull++ [40] to obtain RRE rho values that are given in Tables 5, 6. For Board-B, we always achieve the best fitting with a lognormal distribution for $M_b < 14$ and an exponential distribution for $M_b > 14$. Therefore there is a change point approximately at the 14th month and we determine an exponential distribution for $h_2(t)$. For Board-B, we always achieve the best fitting with a lognormal or a Weibull distribution for $M_b < 18$ and an exponential distribution for $M_b > 18$. Therefore there is a change point approximately at the 18th month and we determine an exponential distribution for $h_2(t)$. Fig. 5 summarizes the results with showing rho versus M_b curves.

After determination of the change point, two phase hazard rate function, consisting of Weibull and exponential distributions, can be constructed by using the following equation:

$$h_o = I(t \leq t_c)\frac{\beta t^{\beta-1}}{\eta^{\beta}} + I(t > t_c)\lambda \tag{10}$$

where β and η are the shape and scale parameters of the Weibull distribution $h_1(t)$, respectively. Additionally, λ is the hazard rate of the exponential distribution $h_2(t)$. Plots of $h_o(t)$, $h_1(t)$, and $h_2(t)$ for Board-B and Board-F are shown in Figs. 6 and 7, respectively. For comparison we also show single Weibull distribution and kernel smoothed hazard rate curves.

Note that throughout this paper, we use "a day" as a unit time interval. Correspondingly all values are derived using day counts. For example, hazard rate values represent failure probabilities in a day; similarly time to failure values are derived in days. However, we sometimes present values in month or year counts just for simplicity.

We successfully apply our method to our four different board data sets. The change point and the distribution parameter values are given in Table 7. The average hazard rate values are consistent with the expectations of the cooperated company that is between 3 and 5 ppm. For all data sets, we determine the change point from the backward analysis and we use Weibull and exponential distributions for modelling. However, these are not necessary and sufficient conditions. For different data sets, the change point could be derived using the forward analysis. Additionally, there

Table 3
Log-likelihood values of different distributions for Board-B in forward direction.

Forward time window	Exponential	Lognormal	Weibull	Gamma
0–30 days (M_f=1)	−18.20	−14.35	−14.14	−14.72
0–60 days (M_f=2)	−17.96	−14.17	−13.92	−14.22
0–90 days (M_f=3)	−16.45	−13.88	−13.73	−13.29
0–180 days (M_f=6)	−16.05	−13.77	−13.57	−13.81
0–270 days (M_f=9)	−14.54	−13.62	−13.42	−13.68
0–360 days (M_f=12)	−14.23	−13.54	−13.34	−13.57
0–540 days (M_f=18)	−13.93	−13.21	−13.26	−13.87
0–720 days (M_f=24)	−13.38	−13.39	−13.20	−14.14
0–900 days (M_f=30)	−13.61	−13.36	−13.15	−13.48
0–1080 days (M_f=36)	−13.42	−13.37	−13.20	−13.34

Table 4
Log-likelihood values of different distributions for Board-F in forward direction.

Forward time window	Exponential	Lognormal	Weibull	Gamma
0–30 days (M_f=1)	−14.80	−13.52	−13.45	−13.44
0–60 days (M_f=2)	−14.34	−13.32	−13.22	−13.40
0–90 days (M_f=3)	−14.25	−13.27	−13.17	−13.35
0–180 days (M_f=6)	−13.84	−13.03	−13.06	−13.25
0–270 days (M_f=9)	−13.52	−13.00	−12.96	−13.09
0–360 days (M_f=12)	−13.50	−12.94	−12.96	−13.09
0–540 days (M_f=18)	−13.37	−12.87	−12.93	−13.05
0–720 days (M_f=24)	−13.18	−12.86	−12.84	−12.98

Table 5
Rho values for of different distributions for Board-B in backward direction.

Backward time window	Exponential	Lognormal	Weibull	Gamma
1080 − 900 days (M_b=30)	0.938	0.931	0.925	0.826
1080 − 720 days (M_b=24)	0.912	0.911	0.904	0.831
1080 − 630 days (M_b=21)	0.903	0.901	0.883	0.818
1080 − 540 days (M_b=18)	0.886	0.856	0.843	0.800
1080 − 450 days (M_b=15)	0.859	0.845	0.833	0.800
1080 − 360 days (M_b=12)	0.721	0.812	0.787	0.731
1080 − 270 days (M_b=9)	0.642	0.788	0.763	0.714
1080 − 180 days (M_b=6)	0.517	0.822	0.765	0.800
1080 − 90 days (M_b=3)	0.612	0.912	0.819	0.826
1080 − 0 days (M_b=0)	0.614	0.911	0.819	0.831

Table 6
Rho values for of different distributions for Board-F in backward direction.

Forward time span	Exponential	Lognormal	Weibull	Gamma
720 − 630 days (M_b=21)	0.956	0.935	0.943	0.922
720 − 540 days (M_b=18)	0.955	0.932	0.930	0.921
720 − 480 days (M_b=16)	0.831	0.922	0.923	0.913
720 − 360 days (M_b=12)	0.793	0.912	0.911	0.902
720 − 270 days (M_b=9)	0.711	0.886	0.845	0.853
720 − 180 days (M_b=6)	0.673	0.882	0.836	0.831
720 − 90 days (M_b=3)	0.622	0.884	0.835	0.822
720 − 0 days (M_b=0)	0.631	0.883	0.832	0.815

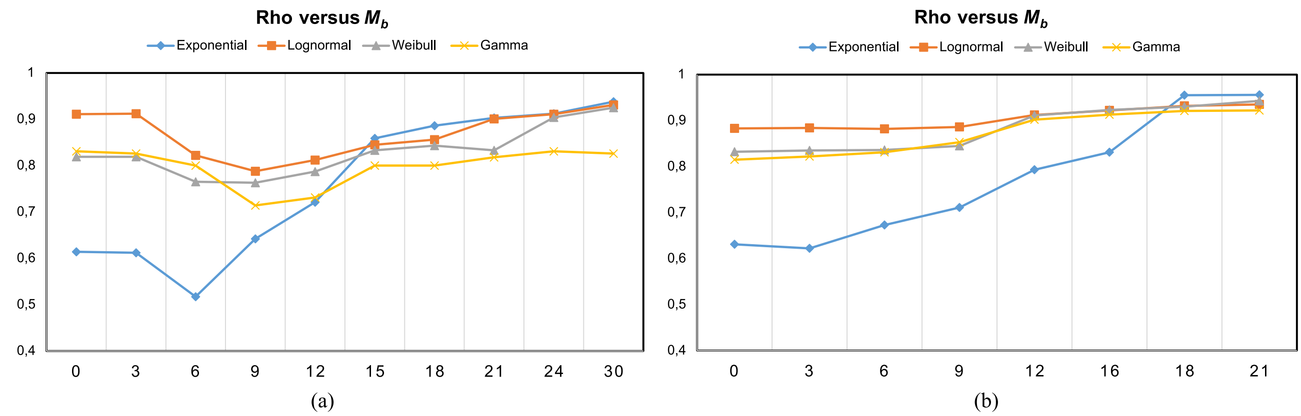

Fig. 5. Rho versus M_b curves of different distributions in backward direction for (a) Board-B and (b) Board-F.

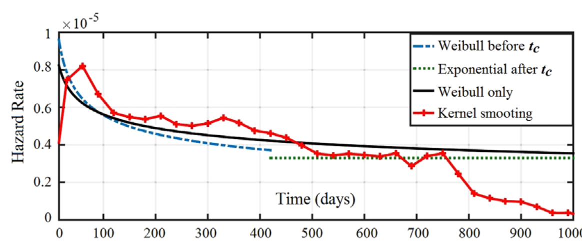

Fig. 6. Hazard rate curves obtained with kernel smoothing, single Weibull distribution, and Weibull-exponential scheme for Board-B.

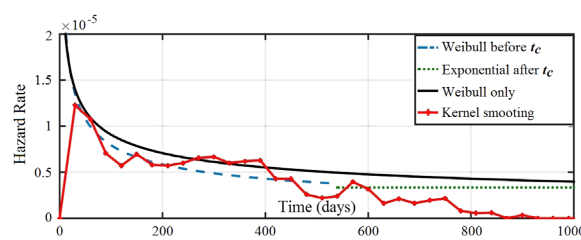

Fig. 7. Hazard rate curves obtained with kernel smoothing, single Weibull distribution, and Weibull-exponential scheme for Board-F.

Table 7
Parameter and change point values obtained by the proposed method.

Board name	β	η (days)	λ (per day)	t_c (months)
Board-B	0.715	2.44×10^6	3.30×10^{-6}	14
Board-E	0.436	1.60×10^9	2.00×10^{-6}	5
Board-F	0.557	1.39×10^7	3.35×10^{-6}	18
Board-K	0.491	1.54×10^3	8.00×10^{-5}	6

can be seen different distributions rather than Weibull and exponential distributions. We see these cases when we apply our method to the heart transplant data set in the following section.

3.2.1. *Application to the heart transplant data set*

Our method is suitable for right censored survival data which is often encountered in reliability engineering and medical applications. After testing our method for the board data sets, here we test it using a well-known heart transplant data. The data has 184 samples and 113 of them are suspended; survival time is 3695 days. Kernel smoothed hazard rate curve of the data is given in Fig. 8. In the literature, the heart transplant data is modelled with two different exponential distributions with a change point. It has been found that the data has a change point between 65 and 80 days with hazard rates (per day) being approximately 0.004 (λ_1) and 0.0004 (λ_2) before and after the change point, respectively

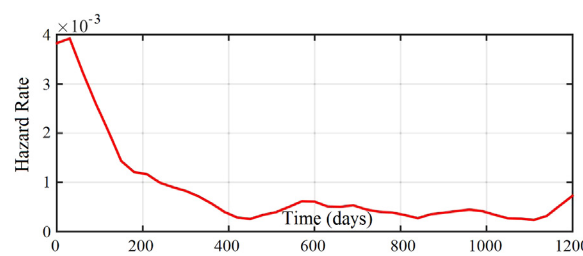

Fig. 8. Kernel smoothed hazard rate curve of the Stanford heart transplant data.

[15,18,41–43]. Therefore the hazard rate function, consisting of exponential and exponential distributions, can be constructed by using the following equation:

$$h_o = I(t \le t_c)\lambda_1 + I(t > t_c)\lambda_2. \qquad (11)$$

We perform our method to find the change point with selecting exponential-exponential hazard rate scheme. Since sample size is relatively small, we use a RRE method to estimate distribution parameters. We observe changes in hazard rate as well as in rho values in forward and backward directions. Obtained values for the forward and backward analysis are given in Tables 8,9, respectively. Analysing the hazard rate values in Table 8, we see that they stand almost steady for the first 90 days and then begin to slowly deviate from 0.0042. Additionally, rho values start to sharply decrease after 80 days. Therefore, we can comment that the change point t_c should be around 80 days. On the other hand, the values in Table 9 do not give us a valuable inference; there is no certain break-away point for the hazard rate or the rho values.

As a result, we find the change point using the forward analysis in an expected range between 65 and 80 days. The reason of the inefficiency of the backward data analysis is that for this data set, in contrary to our board data sets, most of the samples have TTF values larger than the change point. Note that the survival time is

Table 8
Forward analysis of the heart transplant data.

Forward time window	Hazard rate	Rho
0-to-30 days	0.0040	0.979
0-to-60 days	0.0042	0.984
0-to-80 days	0.0042	0.976
0-to-90 days	0.0042	0.947
0-to-100 days	0.0040	0.945
0-to-110 days	0.0039	0.944
0-to-120 days	0.0036	0.944
0-to-150 days	0.0031	0.936
0-to-240 days	0.0023	0.926
0-to-3695 days	0.0007	0.916

Table 9
Backward analysis of the heart transplant data.

Backward time window	Hazard rate λ (per day)	Rho
180-to-3695 days	0.00022	0.940
120-to-3695 days	0.00028	0.951
70-to-3695 days	0.00028	0.961
60-to-3695 days	0.00036	0.912
50-to-3695 days	0.0004	0.931
0-to-3695 days	0.0007	0.916

3695 days. Also note that we only use an exponential-exponential hazard rate scheme considering the same treatment in the related literature. However, using different distributions as we did in the previous section would surely give us additional inferences about the change point.

4. Full warranty reliability prediction

We make a warranty forecasting with a new product's field data that is classified as long-term or short-term by applying the forward and backward data analysis presented in the previous section. If the analysis results in a change point that separates early failure and useful life regions then we call the input data as long-term data. If the analysis results in a single distribution, no change point found, then we call the input data as short-term data. For both data types, modelling is achieved using Weibull and exponential distributions.

In case of having long-term data, reliability prediction is achieved with an exponential distribution that is expected to keep its validity until the end of the warranty period. To elucidate this, we use n-month data of Board-B for which TTF values in months are smaller than n. Of course, if $n=36$ then we have full warranty data for which, as previously found, the change point t_c is 14 months and the hazard rate λ after the change point is 3.3×10^{-6} failures per day. Starting with $n=14$ we increase n. For $n \geq 17$ we can accurately determine the values of t_c and λ regarding that we have adequate number of samples. If $n=17$, $\lambda = 4.2 \times 10^{-6}$ is achieved. As expected increasing n makes the predicted λ get closer to 3.3×10^{-6}. Fig. 9 shows an example for this when $n=20$. Our prediction shown with the dashed red curve covering the warranty period of 20-to-36 months, performs well considering a slight deviation from the blue curve that is obtained using the full warranty data of 36 months. Here, our prediction method is quite simple and straightforward, but we need to wait considerable amount of time after a new product gets into the field. This underlines the importance of using short-term data for early reliability prediction.

In case of having short-term data, we use a framework that connects the prior knowledge based on an old product with a new one. We base our framework on a Weibull distribution and its shape parameter β. We investigate how the β parameter changes by increasing time to failure (TTF) values; we obtain a time dependent two-parameter equation of β. This is thoroughly explained in the following section. In order to estimate other parameters in the Weibull-exponential scheme that are the change point, the Weibull η parameter, and the exponential λ parameter, we use empirical findings and iteration methods.

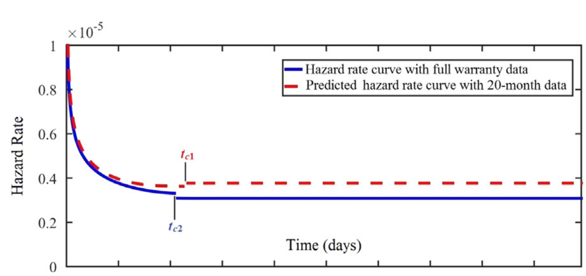

Fig. 9. Comparison of the results obtained from 20-month and 36-month data of Board-B. Using 20-month data: $t_{c1} = 15$, $\lambda_1 = 3.9 \times 10^{-6}$; using 36-month data: $t_{c2} = 14$, $\lambda_2 = 3.3 \times 10^{-6}$.

4.1. Prediction of the Weibull β parameter with short-term data

Considering the fact that different versions of a product used for a specific purpose share similar components and stress mechanisms, valuable inferences for a new product's reliability can be achieved by analysing the reliability performance of old products. In this regard, we classify products into families. Products in the same family have same duties. For example, electronic cards of coffee machines (as opposed to kettles) or electronic cards of car engines (as opposed to truck engines) fall into a same family. For a specific family, we first make a mathematical model of the Weibull β parameter in time domain using full warranty data of an old product. Then we predict a new product's reliability using the developed model and short-term field data of a new product.

We use full warranty data of an old product to derive a time dependent function of β. For this purpose, we find β values for different subsets of the data, namely for different m values such that TTF values less than or equal to m months. Starting with $m=1$, we increase m one by one. We find a β value for each m value and fit the obtained β values using a logarithmic function:

$$\beta(t) = \alpha \times \ln(\zeta t) \tag{12}$$

where t represents the duration of the product in field within its warranty period. Additionally, ζ and α are defined as family and product dependent parameters, respectively. Although we prefer month counts for grouping the data, we use “a day” as a unit time interval (as always in this paper). Therefore, the units of t and ζ are “day” and “1/ day”, respectively; α is unitless.

Using the full warranty data of an old product, we aim to find the ζ parameter to be directly used for new products within the same family. Whenever we have sufficient amount of data for the new product, we can obtain the α parameter and predict the new product's β throughout its warranty period.

In order to elucidate the proposed β prediction method, we apply it to our board data sets. While Board-B, Board-E, and Board-F, used in washing machines, fall into a same family, Board-K used in refrigerators comprises a different family. We can select Board-B, Board-E, or Board-K as an old board since they all have 36-month full warranty data. Table 10 shows β values and corresponding sample sizes (number of failures) for these data sets. We use an MLE method to obtain the β values. Fitting accuracy can be justified using log-likelihood values that are previously presented in Table 3 for Board-B. Similar results are achieved for Board-E and

Table 10
β values & sample sizes for different m values for Board-B, Board-E, and Board-K.

Time	Board-B	Board-E	Board-K
30 days ($m=1$)	0.328, 250	0.332, 528	0.312, 174
60 days ($m=2$)	0.373, 408	0.369, 835	0.362, 370
90 days ($m=3$)	0.421, 538	0.381, 1036	0.413, 601
180 days ($m=6$)	0.471, 827	0.436, 1656	0.496, 1151
270 days ($m=9$)	0.544, 1170	0.489, 2280	0.574, 1697
360 days ($m=12$)	0.613, 1396	0.551, 2845	0.667, 2284
540 days ($m=18$)	0.715, 1669	0.648, 3591	0.812, 3184
720 days ($m=24$)	0.775, 1777	0.699, 3941	0.901, 3687
900 days ($m=30$)	0.797, 1812	0.588, 4046	0.942, 3880
1080 days ($m=36$)	0.800, 1849	0.588, 4050	0.950, 3936

Table 11
ζ, α, and RMSE values for Board-B, Board-E, and Board-K.

Board-B	Board-E	Board-K
ζ=0.941	ζ=0.922	ζ=0.195
α=0.110	α=0.094	α=0.143
RMSE=0.0393	RMSE=0.0652	RMSE=0.0121

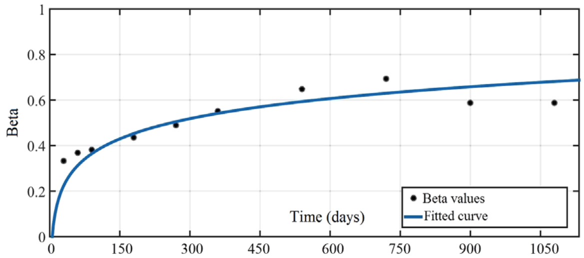

Fig. 10. β values and logarithmic curve fitting for Board-E.

Board-K. After obtaining the β values, we fit them using the logarithmic function in Eq. (12) to find the values of ζ and α. For this purpose, we use a standard least square method; accuracy is evaluated with root-mean-square error (RMSE) values. The obtained ζ, α, and RMSE values are listed in Table 11. Examining the numbers, we see that the ζ values are close for Board-B and Board-E since these boards are in the same family. However, Board-K is in a different family and as expectedly it has a quite different ζ value.

We select Board-E as an old board with 36-month data and aim to predict β values for Board-B, Board-F, and Board-K using their 3-month field data. The fitted β curve of Board-E is shown in Fig. 10. The reason of selecting Board-E as an old board is that it is the oldest board used in the field and it has the largest data size. Indeed, selecting Board-B as an old board is expected to give us similar results since it has almost the same ζ value as shown in Table 11. However, we could not select Board-K as an old board since it is the only member of its family and can not be used for reliability prediction for the other boards.

Using ζ=0.922 for Board-E, we now have a β equation for Board-B and Board-F as

$$\beta(t) = \alpha \times \ln(0.922t). \tag{13}$$

Values of α for Board-B and Board-F are calculated directly from their 3-month data, i.e., TTF ≤ 3 months since by the third month there are usually an adequate number of records or sample sizes to perform an MLE method with reasonable likelihood values. Parameter α is calculated for Board-B as 0.126 and for Board-F as 0.188. Results are shown in Figs. 11 and 12. In these figures, we compare predicted curves that are obtained using 3-month data with discrete points representing β values that are obtained using full warranty data. It is clear that there is a good match between the predicted curves and the points.

An additional result that shows consistency of our method is given in Fig. 13, which shows results for Board-K. The predicted curve of Board-K is obtained following the same process used for other boards with the same value of ζ. There is a substantial difference between real β values and the estimated curve in Fig. 13. This happens mainly because Board-K is not a member of the family of Board-B, Board-E, and Board-F.

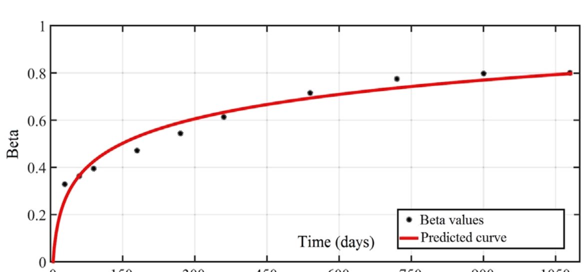

Fig. 11. Comparison of the predicted β curve and the obtained β values for Board-B.

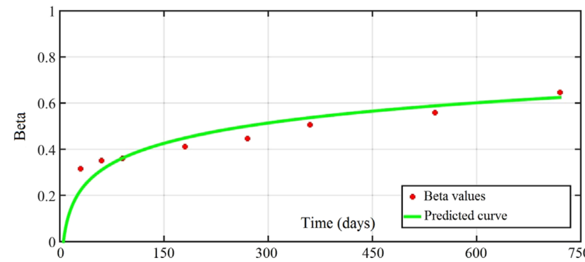

Fig. 12. Comparison of the predicted β curve and the obtained β values for Board-F.

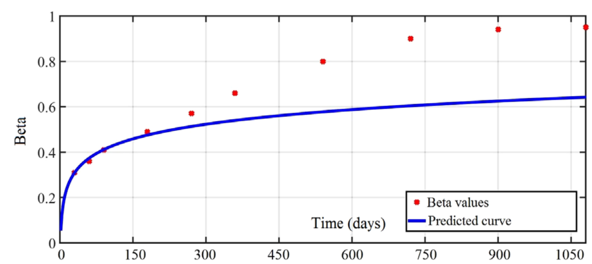

Fig. 13. Comparison of the predicted β curve and the obtained β values for Board-K.

4.2. Prediction of the Weibull-exponential scheme with short-term data

To construct a Weibull-exponential hazard rate scheme of a new product, along with the estimated β parameter we need to find three more parameters that are the change point t_c, the Weibull scale parameter η, and the exponential rate parameter λ. Weibull and exponential hazard rate functions are shown below.

$$h_o(t) = \begin{cases} h_1(t), & t < t_c \\ h_2(t), & t > t_c \end{cases} \tag{14}$$

$$h_1(t) = h_w(t) = \frac{\beta(t_c)}{\eta}\left(\frac{t}{\eta}\right)^{\beta(t_c)-1} \tag{15}$$

$$h_2(t) = h_e(t) = \lambda \tag{16}$$

To find the values of t_c, η, and λ, we use the following three equations. The subscripts *op* and *w* stand for *old product* and *warranty*, respectively; t_w is a warranty period which is 36 months in this study.

$$h_1(t_c) = h_2(t_c) = \lambda \tag{17}$$

$$\lambda = \lambda_{op} \tag{18}$$

$$\frac{1}{t_w - t_c}\int_{t_c}^{t_w} \frac{\beta(t_w)}{\eta}\left(\frac{t}{\eta}\right)^{\beta(t_w)-1} dt$$

$$= \frac{1}{t_w - t_{c-op}}\int_{t_{c-op}}^{t_w} \frac{\beta_{op}(t_w)}{\eta_{op}}\left(\frac{t}{\eta_{op}}\right)^{\beta_{op}(t_w)-1} dt \tag{19}$$

Eq. (17) is a result of the inference that at time t_c, Weibull and exponential hazard rate values should be close to each other. Therefore we assume that they are equal. Eq. (18) and Eq. (19) are

Table 12
Predicted values in the Weibull-exponential scheme using 3-month data. Note that for all boards $\lambda = \lambda_{op} = 2.4 \times 10^{-6}$ (per day).

Board name	β	η (days)	t_c (months)
Board-B	0.684	3.3×10^6	17
Board-F	0.6	8.1×10^6	20
Board-K	0.6	3.3×10^6	16

based on our assumption that products in the same family have a similar reliability performance in their useful life region. More generally stating, although different versions of a product might perform substantially differently in their early failure region, they have almost identical behaviour for their useful life region. Indeed, this is a well-known assumption frequently used by reliability scientist/engineers in the industry. We can justify it with our board data sets, by considering the λ values for Board-B, Board-E, and Board-F presented in Table 7; there is nearly a $\pm 20\%$ dispersion. Similarly, a dispersion of $\pm 10\%$ is seen for a different product group [11].

In Eq. (18), we assume that the λ value of a new product is same as that of an old product. In Eq. (19), by using a Weibull distribution throughout the whole warranty period, we assume that the average hazard rate value from the change point to the end of the warranty of a new product is same as that of an old product. With using Eq. (14) through Eq. (19) and performing an empirical iteration, we find values of t_c, η, and λ of our board data sets as shown in Table 12. For Board-B and Board-F, the predicted values, using 3-month warranty data, are in consistent with the values obtained using full warranty data in Table 7. However, the predicted values for Board-K are quite different than the values in Table 7 since Board-K is not in not a member of the family of Board-B, Board-E, and Board-F.

For further evaluation of our prediction method, Figs. 14 and 15 are presented with predicted and real curves where solid and dashed lines are used to model early failure and useful life regions, respectively. The figures confirm our method's accuracy; predicted and real curves are satisfactorily matched with slight differences between t_c values (intersection of the solid and dashed lines), curve shapes (determined by β), and average hazard rate values (determined by β and η). Note that our prediction method is fundamentally based on our assumption that products in the same family have a similar reliability performance in their useful life region. Here, to predict η, one can consider applying a similar procedure as we do for β, but this is certainly not a good idea since η is a scale parameter representing the time when 63% of the products fail. Therefore, its value is expected to highly fluctuate by changing a product type and/or the field data duration.

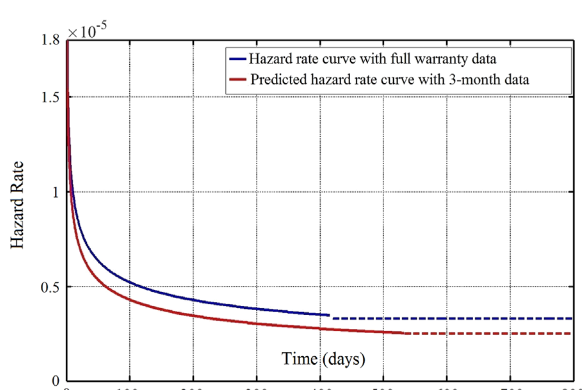

Fig. 14. Hazard rates for Board-B using 3-month and 36-month full warranty data.

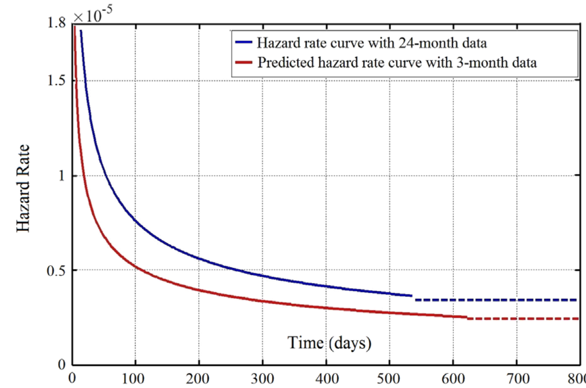

Fig. 15. Hazard rates for Board-F using 3-month and 24-month data.

5. Merits, limitations, and future work

In this study, we aim to predict reliability performance of high-volume electronic products throughout their warranty period by using field return data. We construct our prediction model on a Weibull-exponential hazard rate scheme by using the proposed change point detection method. Conventional reliability prediction methods can only predict very near future reliability of a product since they assume that failure mechanisms of a product do not change. In this study, we propose a new method to overcome this problem. The proposed method is evaluated by using four different data sets corresponding to four different boards. Warranty period of these boards is 3 years, and we perform prediction by using their field data as short as 3 months. The predicted results from our method and the direct results from the field return data matches well. This demonstrates the accuracy of our model.

In developing our prediction methodology, we aim to keep balance between its technical contribution and practicability. As opposed to use purely statistical methods, we aim to combine statistical, empirical, and engineering point of views that results in applicability of our methodology to different products not just limited to electronic products. However, we have three main limitations. First, the product's working environments and stress levels should be satisfactorily steady. For example, consider a vehicle engine/motor working in four seasons with a temperature range from $-20\,°C$ to $20\,°C$. For this product we can not use its short-term data corresponding to a winter season to predict the product's one-year reliability performance. We consider this limitation and its solutions as a future work.

Second, in order to apply our β prediction method, the product should have different parts/components having different β values. Ideally, a specific component or a material is expected to have its own constant β that is time independent. However, similarly for electronic products studied in this work, if the product has several components having different β values larger and smaller than 1 then we can apply our method. Third, the product should have sufficient amount of field data to successfully apply an MLE method for both change point detection and reliability prediction steps. In this regard, military and aerospace products are not good candidates for our model. Indeed, in general low sample sizes are problematic for reliability statistics. To overcome this problem, different estimation methods including a Bayesian estimation method, can be integrated into our model as a future work.

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